Armand Borel

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Armand Borel (* 21 May 1923, La Chaux-de-Fonds, Swiss – + 11 August 2003, Princeton, New Jersey, USA) was a Swiss mathematician, attended secondary school in Geneva but was also educated at a number of private schools. In 1942, at the age of nineteen, he entered the École Polytechnique Fédérale in Zürich where he studied mathematics and physics. After completing military service, which was compulsory in Switzerland, he graduated with a diploma in mathematics in 1947 having undertaken his master's studies under Eduard Stiefel. As well as Stiefel, Borel had attended lectures at the École Polytechnique Fédérale by Hopf who played an important role in influencing Borel's mathematical tastes. Following his graduation, Borel was appointed as a teaching assistant at the École Polytechnique Fédérale in Zürich. His aim was to undertake research for his thesis on Lie groups and during his two years as a teaching assistant he published two papers on the topic. However, receiving an exchange grant from the French Centre National de la Recherche Scientifique he was able to spend the year 1949-50 in Paris. This was extremely important for him for he was able to get to know, and to learn from, Henri Cartan, Jean Dieudonné, and Laurent Schwartz. He made friends with younger mathematicians, Roger Godement, Pierre Samuel, Jacques Dixmier, and especially with Jean-Pierre Serre. Jean Leray became Borel's thesis supervisor and he attended courses which he gave at the Collège de France. Borel wrote [6]:

All these people – the elder ones, of course, but also the younger ones – were very broad in their outlook. They knew so much and knew it so well. They shared an efficient way to digest mathematics, to go to the essential points, and reformulate the mathematics in a more comprehensive and conceptual way. Even when discussing a topic more familiar to me than to them, their sharp questions often gave me the impression I had not really thought it through. That methodology was also apparent in some of the lectures at the Bourbaki seminar, such as Weil's on theta functions or Schwartz's on Kodaira's big Annals paper on harmonic integrals.

After his year in Paris, Borel went to Geneva where he substituted for the professor of algebra from 1950 to 1952. However, during this time he made frequent visits to both Zurich and to Paris. In the summer of 1951 he gave a series of lectures in Zürich on the Leray's ideas on the theory of homological invariants of locally compact spaces and of continuous mappings.
which was published as a 95 page book of mimeographed notes with the title Cohomologie des espaces localement compacts, d'après J Leray. During the whole period when he was based in Geneva he was working on his thesis on the cohomology with integer coefficients of Lie groups which he defended at the Sorbonne in Paris in the early part of 1952.

In 1952 Borel married Gabrielle Aline Pittet; they had two daughters Dominique Odette Susan and Anne Christine. In the autumn of 1952 Borel, and his new wife Gaby (as she was always known) set off for the United States. Borel had been invited to spend a year at the Institute for Advanced Study at Princeton and this was extended to a second year. Haefliger writes [8]:

This two year stint in Princeton would prove crucial to the broadening of his mathematical interests ... [in a paper] he describes with enthusiasm the exhilarating mathematical ambiance of the time.

Leaving Princeton in 1954, he spent the academic year at Chicago where André Weil was teaching. This was an opportunity for Borel to learn a great deal about algebraic geometry and number theory from Weil. He returned to the country of his birth in 1955 when he was appointed professor of mathematics at the École Polytechnique Fédérale in Zurich. Happy to be back in his native land, he had to make a difficult decision when he was offered a permanent professorship at the Institute for Advanced Study at Princeton in the following year. After taking some time to reach the difficult decision on whether to accept this highly prestigious offer, he made his decision and took up his duties in the United States in the spring of 1957.

We should note at this point the major contribution that Borel made to Bourbaki. He describes his experiences in [6] but to a certain extent plays down the major role he had in writing the nine Bourbaki chapters on Lie groups and Lie algebras which are considered now to be perhaps the most valuable long-lasting contribution of Bourbaki. Borel writes in [6]:

It started with a draft of about 70 pages on root systems. [I] was almost apologetic in presenting to Bourbaki such a technical and special topic, but asserted this would be justified later by many applications. When the next draft, of some 130 pages, was submitted, one member remarked that it was all right, but really Bourbaki was spending too much time on such a minor topic, and others acquiesced. Well, the final outcome is well known: 288 pages, one of the most successful books by Bourbaki. It is a truly collective work, involving very actively about seven of us, none of whom could have written it by himself.

Pierre Cartier writes [12]:
If you look at the volumes on Lie groups, you will see that the later ones have chapters that you don't expect in Bourbaki. It became more and more explicit; there are tables and drawings. I think this was basically the influence of one person, Armand Borel. He was fond of quoting Shaw, "It's the Swiss national character, my dear lady," and very often during a discussion he would say, "I'm the Swiss peasant."

Haefliger sums up his contributions in [8]:

His body of mathematical work is considerable and shows remarkable coherence. Borel's work, apart from a dozen books, lecture notes ...encompasses more than 150 articles. Beyond 50 of them are written in collaboration with more than 30 coauthors (notably ten joint works with J-P Serre and five with J Tits). They focus on Lie groups, and their actions, as well as on algebraic and arithmetic groups, and broach core questions regarding many different areas: algebraic topology, differential geometry, analytic geometry, analytic and algebraic geometry, number theory etc. These articles were fundamental to the development of mathematics in the second half of the 20th century.

Among his books are Topics in the homology theory of fibre bundles (1967), which is based on lectures Borel gave at the University of Chicago in 1954 in which he described the state of the topic at that time adopting the same methods and points of view as in his thesis. After 1955, Professor Borel turned to algebraic groups, producing the classic paper Groupes lineaires algebriques, which represented a turning point in the history of the subject and which led to many important developments. At the same time, he was studying, and eventually solving (with H. Chandra and A. Baily), some of the most basic and difficult problems of the theory of arithmetic groups. Professor Borel also worked on the cohomology of arithmetic groups and its applications, as well as various aspects of new cohomological theories, automorphic forms, and the infinite-dimensional representation theory of real and p-adic Lie groups. In 1969 he published Introduction aux groupes arithmétiques which was also based on a lecture course, this time at the Institut Henri-Poincaré in 1967. Also in 1969 Linear algebraic groups was published based on a graduate course given by Borel at Columbia University in the spring of 1968. One book which does not seem to be based on a lecture course is Automorphic forms on SL(R) which Borel himself says would have been better titled Introduction to some aspects of the analytic theory of automorphic forms on SL(R) and the upper half-plane X.

Borel received many honours for his outstanding contributions to mathematics. He was awarded an honorary degree from the University of Geneva in 1972, received the Brouwer Medal by the Dutch Mathematical Society in 1978, was elected to the American Academy of
Arts and Sciences in 1976, the National Academy of Sciences (United States) in 1987, and the Academy of Sciences (Paris) in 1987. He was also elected to the Finnish Academy of Sciences and Letters and the American Philosophical Society. He received the American Mathematical Society's Steele Prize for lifelong contributions to mathematics in 1991. The citation states that Borel's results:

... provided the empirical base for a great swath of modern mathematics, and his observations pointed out the structures and mechanisms that became central concerns of mathematical activity. In the course of amassing these astounding achievements he placed the facilities of the Institute for Advanced Study at the service of mathematics and mathematicians, using them to foster talent, share his ideas, and facilitate access to recent developments through seminars and lectures. It is just simply not possible to cite a career more accomplished or fruitful or one more meaningful to the contemporary mathematical community.

Borel also received the Balzan prize in 1992:

For his fundamental contributions to the theory of Lie groups, algebraic groups and arithmetic groups, and for his indefatigable action in favour of high quality in mathematical research and of the propagation of new ideas.

In fact we learn much of Borel's view of mathematics in the reply he made on receiving the Balzan prize:

Mathematics is a gigantic intellectual construction, very difficult, if not impossible, to view in its entirety. Sometimes I like to compare it to an iceberg because it has a small visible part and a large invisible part. By visible part I mean the mathematics useful in the external world, in technology, physics, natural sciences, astronomy, computers and so on, whose usefulness and social justification cannot be doubted. Indeed, it is certain that practical problems in ancient times were at the very origin of mathematics. However, with the development of mathematics the subject acquired a life of its own and mathematics became more and more interested in purely mathematical problems, not necessarily paying attention to applications outside mathematics itself. This forms the invisible part of the iceberg; I mean invisible or at least very difficult to grasp for the non-mathematician ...

Mathematics has been for me a profession but also my hobby. The course my investigations have taken, the choice of arguments to study, have been influenced by both points of view, which are often not quite distinct. Again and again I have been by a sense of the architecture of this building to which we continue to add new wings and new floors, while renovating the parts already built ... This is the professional view, but happily these problems were those that
attracted me the most. In other instances I was not guided by such motives, being attracted only by curiosity, by the need to know the answer to an enigma ...

On another occasion he said:

... mathematics is an extremely complex creation, which exhibits so many common traits from art and from both the experimental and theoretical sciences. It reflects simultaneously all three of them and therefore must be distinguished from all three of them.

Among his interests we mention music in particular [5]:

His love of jazz kindled in him an interest in Carnatic music, with its syncopated rhythms and melodic improvisations, which grew into a passion.

Also, as Bombieri writes in [5]:

He loved nature, and quite often I walked with him in the Institute woods ...

We now look at some comments on his personality. Chandrasekharan writes [5]:

His brilliance is in his refusal to distinguish between fun and learning.

Chandrasekharan also writes:

His personality might have seemed dour to those who did not know him well; they could not sense the soft core underneath, nourished and sustained by his devoted wife, Gabrielle. He had a social conscience and human sympathy for the predicament of the poor and disadvantaged.

Prasad writes in the same article:

Borel was an astute observer: he had an uncanny eye for artistic detail and would reflect on the influence of literature and culture on human outlook.

Borel loved to travel and made visits to many countries including India, Mexico and China. He accepted a professorship at ETH in Zurich which he held from 1983 to 1986. His life fell into a regular pattern during his last few years when he would spend the winters in his home at Princeton, spend the spring in the Far East, then go to his home La Conversion overlooking Lac Léman in Switzerland. His final illness was brief and he died after the rapid progression of cancer.

Abstract

Armand Borel (* 21 May 1923, La Chaux-de-Fonds, Switzerland – + 11 August 2003, Princeton, New Jersey, USA) was a Swiss mathematician, and was a permanent professor at the Institute for Advanced Study in Princeton, New Jersey, United States from 1957 to 1993. He worked in algebraic topology, in the theory of Lie groups, and was one of the creators of the contemporary theory of linear algebraic groups. (He used to answer the question of whether he was related to Émile Borel alternately by saying he was a nephew, and no relation.)

He studied at the ETH Zürich. He came under the influence of the topologist Heinz Hopf, and the Lie group theorist Stiefel. He was in Paris from 1949 – he applied the Leray spectral sequence to the topology of Lie groups and their classifying spaces, under the influence of Leray and Henri Cartan.

He collaborated with Jacques Tits in fundamental work on algebraic groups, and with Harish-Chandra on their arithmetic subgroups. In an algebraic group G a Borel subgroup H is one such that the homogeneous space G/H is a projective variety, and as small as possible. The Borel–Moore homology theory applies to general locally compact spaces, and is closely related to sheaf theory. He published a number of books, including work on the history of Lie groups.
References

Books:


Articles:


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