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Katedra statistiky a pravděpodobnosti

STATISTIKA

VZORCE PRO 4ST231

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Popisná statistika

$$p_i = \frac{n_i}{n} \quad \sum_{i=1}^k n_i = n \quad \sum_{i=1}^k p_i = 1 \quad i = 1, 2, \dots, k$$

$$\tilde{x}_p \quad n \cdot \frac{P}{100} < z_p < n \cdot \frac{P}{100} + 1 \quad n p < z_p < n p + 1$$

$$n \frac{P}{100} + 0,5 = z_p \quad n p + 0,5 = z_p$$

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

$$\bar{x} = \frac{\sum_{i=1}^k x_i n_i}{\sum_{i=1}^k n_i}$$

$$\bar{x} = \sum_{i=1}^k x_i p_i$$

$$\bar{x}_H = \frac{n}{\sum_{i=1}^n \frac{1}{x_i}}$$

$$\bar{x}_H = \frac{\sum_{i=1}^k n_i}{\sum_{i=1}^k \frac{n_i}{x_i}}$$

$$\bar{x}_H = \frac{1}{\sum_{i=1}^k \frac{p_i}{x_i}}$$

$$\bar{x}_G = \sqrt[n]{\prod_{i=1}^n x_i} = \sqrt[n]{x_1 \cdot x_2 \cdot \dots \cdot x_n}$$

$$\bar{x}_G = \sqrt[n]{\prod_{i=1}^k x_i^{n_i}} = \sqrt[n]{x_1^{n_1} \cdot x_2^{n_2} \cdot \dots \cdot x_k^{n_k}}$$

$$\bar{x}_K = \sqrt{\frac{\sum_{i=1}^n x_i^2}{n}}$$

$$\bar{x}_K = \sqrt{\frac{\sum_{i=1}^k x_i^2 n_i}{\sum_{i=1}^k n_i}}$$

$$\bar{x}_K = \sqrt{\sum_{i=1}^k x_i^2 p_i}$$

$$R = x_{\max} - x_{\min}$$

$$s_x^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$s_x^2 = \overline{x^2} - \bar{x}^2 = \frac{\sum_{i=1}^n x_i^2}{n} - \left(\frac{1}{n} \sum_{i=1}^n x_i \right)^2$$

$$s_x^2 = \frac{\sum_{i=1}^k (x_i - \bar{x})^2 n_i}{\sum_{i=1}^k n_i}$$

$$s_x^2 = \overline{x^2} - \bar{x}^2 = \frac{\sum_{i=1}^k x_i^2 n_i}{\sum_{i=1}^k n_i} - \left(\frac{\sum_{i=1}^k x_i n_i}{\sum_{i=1}^k n_i} \right)^2$$

Popisná statistika

$$s_x^2 = \sum_{i=1}^k (x_i - \bar{x})^2 p_i$$

$$s_x^2 = \overline{x^2} - \bar{x}^2 = \sum_{i=1}^k x_i^2 p_i - \left(\sum_{i=1}^k x_i p_i\right)^2$$

$$s_x^2 = \overline{s^2} + s_{\bar{x}}^2 = \frac{\sum_{i=1}^k s_i^2 n_i}{\sum_{i=1}^k n_i} + \frac{\sum_{i=1}^k (\bar{x}_i - \bar{x})^2 n_i}{\sum_{i=1}^k n_i} \qquad \bar{x} = \frac{\sum_{i=1}^k \bar{x}_i n_i}{\sum_{i=1}^k n_i}$$

$$s_x^2 = \sum_{i=1}^k s_i^2 p_i + \sum_{i=1}^k (\bar{x}_i - \bar{x})^2 p_i \qquad \bar{x} = \sum_{i=1}^k \bar{x}_i p_i$$

$$s_x = \sqrt{s_x^2} \qquad V_x = \frac{s_x}{\bar{x}}$$

Analýza závislosti

$$S_y = \sum_{i=1}^k \sum_{j=1}^{n_i} (y_{ij} - \bar{y})^2 = S_{y,m} + S_{y,v} \quad P^2 = \frac{S_{y,m}}{S_y} \quad P = \sqrt{P^2}$$

regresní přímka $y = \beta_0 + \beta_1 x + \varepsilon$ $Y = b_0 + b_1 x$ minimum $_{b_0, b_1} \sum_{i=1}^n (y_i - b_0 - b_1 x_i)^2$

$$s_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{n} = \overline{xy} - \bar{x} \cdot \bar{y}$$

$$b_1 = b_{yx} = \frac{n \sum y_i x_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2} = \frac{\overline{xy} - \bar{x} \cdot \bar{y}}{\overline{x^2} - \bar{x}^2} = \frac{s_{xy}}{s_x^2}$$

$$b_0 = \frac{\sum y_i \sum x_i^2 - \sum y_i x_i \sum x_i}{n \sum x_i^2 - (\sum x_i)^2} = \bar{y} - b_{yx} \bar{x}$$

$$r_{yx} = r_{xy} = \frac{n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \sum_{i=1}^n y_i}{\sqrt{n \sum_{i=1}^n x_i^2 - (\sum_{i=1}^n x_i)^2} \sqrt{n \sum_{i=1}^n y_i^2 - (\sum_{i=1}^n y_i)^2}} = \frac{\overline{xy} - \bar{x} \bar{y}}{\sqrt{(\overline{x^2} - \bar{x}^2)(\overline{y^2} - \bar{y}^2)}} = \frac{s_{xy}}{s_x s_y}$$

Jiné regresní funkce

$$Y = b_0 + b_1 x + b_2 x^2 \quad Y = b_0 + b_1 \frac{1}{x} \quad Y = b_0 + b_1 \log x$$

$$Y = b_0 + b_1 x_1 + b_2 x_2 + \dots + b_k x_k$$

$$Y = b_0 b_1^x$$

$$S_y = S_Y + S_{(y-Y)}$$

$$s_y^2 = s_Y^2 + s_{(y-Y)}^2$$

$$S_y = \sum_{i=1}^n (y_i - \bar{y})^2$$

$$S_Y = \sum_{i=1}^n (Y_i - \bar{y})^2$$

$$S_{(y-Y)} = \sum_{i=1}^n (y_i - Y_i)^2 = \sum_{i=1}^n e_i^2$$

$$s_y^2 = \frac{1}{n} \sum_{i=1}^n (y_i - \bar{y})^2 = \frac{S_y}{n}$$

$$s_Y^2 = \frac{1}{n} \sum_{i=1}^n (Y_i - \bar{y})^2 = \frac{S_Y}{n}$$

$$s_{(y-Y)}^2 = \frac{1}{n} \sum_{i=1}^n (y_i - Y_i)^2 = \frac{S_{(y-Y)}}{n}$$

$$I_{yx}^2 = \frac{S_Y}{S_y} = 1 - \frac{S_{(y-Y)}}{S_y}$$

$$I_{yx} = \sqrt{I_{yx}^2}$$

$$r_{i_x i_y} = 1 - \frac{6 \sum (i_x - i_y)^2}{n(n^2 - 1)}$$

Časové řady

$$\bar{y} = \frac{\sum_{t=1}^n y_t}{n}$$

$$\bar{y} = \frac{\frac{1}{2}y_1 + \sum_{t=2}^{k-1} y_t + \frac{1}{2}y_k}{k-1} \quad \bar{y} = \frac{\frac{y_1+y_2}{2}d_1 + \frac{y_2+y_3}{2}d_2 + \dots + \frac{y_{n-1}+y_n}{2}d_{k-1}}{d_1+d_2+\dots+d_{k-1}}$$

$$\Delta_t^1 = y_t - y_{t-1} \quad \bar{\Delta}^1 = \frac{1}{n-1} \sum_{t=2}^n \Delta_t^1 = \frac{y_n - y_1}{n-1}$$

$$\Delta_t^2 = \Delta_t^1 - \Delta_{t-1}^1$$

$$k_t = \frac{y_t}{y_{t-1}} \quad \bar{k} = \sqrt[n-1]{k_2 k_3 \dots k_n} = \sqrt[n-1]{\frac{y_n}{y_1}}$$

$$y_t = T_t + S_t + C_t + \varepsilon_t$$

$$y_t = T_t S_t C_t \varepsilon_t$$

Trendová složka

$$m = 2p + 1$$

$$\bar{y}_t = \frac{\sum_{i=-p}^p y_{t+i}}{m} = \frac{y_{t-p} + y_{t-p+1} + \dots + y_{t-1} + y_t + y_{t+1} + \dots + y_{t-p+1} + y_{t+p}}{m}$$

$$m = 2p$$

$$\bar{y}_t = \frac{1}{2m} (y_{t-p} + 2y_{t-p+1} + \dots + 2y_{t-1} + 2y_t + 2y_{t+1} + \dots + y_{t-p+1} + y_{t+p})$$

$$T_t = \beta_0 + \beta_1 t$$

$$\hat{T}_t = b_0 + b_1 t$$

$$T_t = \beta_0 + \beta_1 t + \beta_2 t^2$$

$$\hat{T}_t = b_0 + b_1 t + b_2 t^2$$

$$T_t = \beta_0 t^{\beta_1}$$

$$\ln T_t = \ln \beta_0 + \beta_1 \ln t$$

$$\ln(\hat{T}_t) = \ln b_0 + b_1 \ln t$$

$$\text{MSE} = \frac{1}{n} \sum_{t=1}^n (y_t - T_t)^2$$

Sezónní složka

Regresní metoda s umělými proměnnými (lineární trend, délka sezónní periody r)

$$y_{it} = T_{it} + S_{it} + \varepsilon_{it} = \beta_0 + \beta_1 t + \alpha_1 x_{1t} + \alpha_2 x_{2t} + \dots + \alpha_{r-1} x_{r-1t} \quad i = 1, 2, \dots, r \quad r \text{ délka sezónní periody}$$

$$t = 1, 2, \dots, n$$

$$Y_{it} = \hat{T}_{it} + \hat{S}_{it} = b_0 + b_1 t + a_1 x_{1t} + a_2 x_{2t} + \dots + a_{r-1} x_{r-1t}$$

Časové řady

$$\hat{T}_t = (b_0 + \bar{a}) + b_1 t$$

$$\hat{S}_{it} = \hat{S}_i \quad \sum_{i=1}^r \hat{S}_i = 0$$

$$\hat{S}_i = a_i - \bar{a} \quad i = 1, 2, \dots, r-1$$

$$\hat{S}_r = -\bar{a}$$

$$\bar{a} = \frac{\sum_{i=1}^{r-1} a_i}{r}$$

Metoda empirických indexů (délka sezónnosti r)

$$y_{it} = T_{it} C_{it} S_{it} \varepsilon_{it}$$

$$\hat{S}_{it} = \hat{S}I_i \quad \sum_{i=1}^r \hat{S}I_i = r$$

$$\hat{S}I_i = \overline{S}I_i \frac{r}{\sum_{i=1}^r \overline{S}I_i} \quad \text{kde } \overline{S}I_i = \frac{\text{Součet } S_{it} \text{ pro } i\text{-tou sezónu}}{\text{Počet } S_{it} \text{ pro } i\text{-tou sezónu}}$$

$$S_{it} = \frac{y_{it}}{\bar{y}_t}, \quad \bar{y}_t \text{ je klouzavý průměr pro } m = r$$

Indexní analýza

$$I_{t/1} = \frac{y_t}{y_1} = I_{2/1} \cdot I_{3/2} \cdots I_{t/t-1}$$

$$I_{t/t-1} = \frac{y_t}{y_{t-1}} = \frac{I_{t/1}}{I_{t-1/1}}$$

$$Q = pq$$

$$IQ = \frac{Q_1}{Q_0} \quad \Delta Q = Q_1 - Q_0 \quad Ip = \frac{p_1}{p_0} \quad \Delta p = p_1 - p_0 \quad Iq = \frac{q_1}{q_0} \quad \Delta q = q_1 - q_0$$

$$\sum Q = \bar{p} \cdot \sum q$$

$$I(\sum Q) = I\bar{p} \cdot I(\sum q)$$

$$I(\sum Q) = \frac{\sum Q_1}{\sum Q_0} = \frac{\sum p_1 q_1}{\sum p_0 q_0} = \frac{\sum IQ \cdot Q_0}{\sum Q_0} = \frac{\sum Q_1}{\sum \frac{Q_1}{IQ}} \quad \Delta(\sum Q) = \sum Q_1 - \sum Q_0$$

$$I\bar{p} = \frac{\bar{p}_1}{\bar{p}_0} = \frac{\frac{\sum Q_1}{\sum q_1}}{\frac{\sum Q_0}{\sum q_0}} = \frac{\sum p_1 q_1}{\sum p_0 q_0} = \frac{\frac{\sum Q_1}{p_1}}{\frac{\sum Q_0}{p_0}} \quad \Delta\bar{p} = \bar{p}_1 - \bar{p}_0 = \frac{\sum p_1 q_1}{\sum q_1} - \frac{\sum p_0 q_0}{\sum q_0}$$

$$I(\sum q) = \frac{\sum q_1}{\sum q_0} = \frac{\sum Iq \cdot q_0}{\sum q_0} = \frac{\sum q_1}{\sum \frac{q_1}{Iq}} \quad \Delta(\sum q) = \sum q_1 - \sum q_0$$

$$\bar{p} = p \cdot \frac{q}{\sum q}$$

$$I_{\bar{p}} = \frac{\bar{p}_1}{\bar{p}_0} = \frac{\frac{\sum p_1 q_0}{\sum q_0} \cdot \frac{\sum p_1 q_1}{\sum p_1 q_0}}{\frac{\sum p_0 q_0}{\sum q_0} \cdot \frac{\sum p_0 q_1}{\sum p_0 q_0}} \quad \text{nebo} \quad I_{\bar{p}} = \frac{\bar{p}_1}{\bar{p}_0} = \frac{\frac{\sum p_0 q_1}{\sum q_1} \cdot \frac{\sum p_1 q_1}{\sum q_1}}{\frac{\sum p_0 q_0}{\sum q_0} \cdot \frac{\sum p_0 q_1}{\sum q_1}}$$

$$I_{\bar{p}} = I_{SS}^{\left(\frac{q_0}{\sum q_0}\right)} \cdot I_{STR}^{(p_1)} \quad I_{\bar{p}} = I_{STR}^{(p_0)} \cdot I_{SS}^{\left(\frac{q_1}{\sum q_1}\right)}$$

Indexní analýza

$$\sum Q = \sum p \cdot q$$

$$I(\Sigma Q) = \frac{\sum Q_1}{\sum Q_0} = \frac{\sum p_1 q_1}{\sum p_0 q_0} = \frac{\sum IQ \cdot Q_0}{\sum Q_0} = \frac{\sum Q_1}{\sum \frac{Q_1}{IQ}} \quad \Delta(\Sigma Q) = \sum p_1 q_1 - \sum p_0 q_0$$

$$I(\sum Q) = \frac{\sum Q_1}{\sum Q_0} = \frac{\sum p_1 q_1}{\sum p_0 q_0} = \frac{\sum p_0 q_1}{\sum p_0 q_0} \cdot \frac{\sum p_1 q_1}{\sum p_0 q_1} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \cdot \frac{\sum p_1 q_1}{\sum p_1 q_0}$$

$$= I_q^{(L)} \cdot I_p^{(P)} = I_p^{(L)} \cdot I_q^{(P)}$$

$$I_p^{(L)} = \frac{\sum p_1 q_0}{\sum p_0 q_0} = \frac{\sum Ip \cdot p_0 q_0}{\sum p_0 q_0} = \frac{\sum Ip \cdot Q_0}{\sum Q_0} \quad I_p^{(P)} = \frac{\sum p_1 q_1}{\sum p_0 q_1} = \frac{\sum p_1 q_1}{\sum \frac{p_1 q_1}{Ip}} = \frac{\sum Q_1}{\sum \frac{Q_1}{Ip}}$$

$$I_p^{(F)} = \sqrt{I_p^{(L)} \cdot I_p^{(P)}}$$

$$I_q^{(L)} = \frac{\sum p_0 q_1}{\sum p_0 q_0} = \frac{\sum Iq \cdot p_0 q_0}{\sum p_0 q_0} = \frac{\sum Iq \cdot Q_0}{\sum Q_0} \quad I_q^{(P)} = \frac{\sum p_1 q_1}{\sum p_1 q_0} = \frac{\sum p_1 q_1}{\sum \frac{p_1 q_1}{Iq}} = \frac{\sum Q_1}{\sum \frac{Q_1}{Iq}}$$

$$I_q^{(F)} = \sqrt{I_q^{(L)} \cdot I_q^{(P)}}$$